Multiparameter Adaptive Process Control via Constrained Objective Functions

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A method has been developed for applying classical minimization techniques to forms of algebraic performance indices for use in optimal adaptive control systems. Essentially, the derivative of a general objective function is constrained to be equal to the integrand of a desired integral performance index. This generates a set of linear algebraic equations, the solution of which results in an algebraic objective function which is explicit in the system parameters. Derivatives of this function can be taken with respect to the controllable parameters, set equal to zero, and solved for the settings which minimize the performance index over time. Alternatively, the function itself may be searched for its minimum on the controllable parameters.

The method has been applied to a stirred tank chemical reactor with an exothermic first-order reaction in which heat removal was accomplished by cooling coils. The cooling water flow rate was controlled by a proportional-plus-integral controller and by a three-mode controller. The adaptive control system adjusted the controller settings periodically. The plant, which was third-order, was controlled to a second-order dynamic reference model. The responses to both initial offsets (start-up problem) and disturbances in system parameters were investigated. In all cases, the adaptive control system performance was markedly superior to that for the unadapted, ordinary feedback control system.

Chemical engineers in recent years have made significant advances in describing physical phenomena which are of interest to the process industries, but advances in the control of these processes have not always kept pace primarily because of the need for expensive hardware to implement the control. Recent developments in small, relatively inexpensive computers are encouraging greater use of sophisticated control schemes since the cost of implementation has been lowered. In particular, adaptive control is finding promising applications in the process control field, because it has important advantages over ordinary feedback control.

Many definitions have been proposed for adaptive control (*2* to *4*, *9*). In this work a control feature will be considered to be adaptive if, rather than adjusting independent input variables, it adjusts parameters which directly affect system dynamic characteristics. This is equivalent to varying coefficients in homogeneous system differential equations as opposed to specifying forcing functions in the complete system equations. These adjustments are made in the face of unknown disturbances based on a comparison of the current state of the system with its desired state.

MODEL REFERENCE

A block diagram of a model reference adaptive control system is given in Figure 1. Adaptive control is being exercised over controller parameters, such as a proportionality constant or reset rate. These parameters are adjusted after a decision based on a comparison of the model output and plant output. Note that the model has been subjected to the same inputs as the plant, but has not been subjected to the unpredictable and uncontrollable disturbances which affect the plant. This type of adaptive control is called model reference, as the decision on how to adjust parameters is based on the deviations of the plant output from the model output.

The model used may be static or dynamic at the discretion of the system designer. If a static model is chosen, control to a constant output results. Static models would result from an attempt to control the plant output to a fixed value in face of external disturbances. If the model is dynamic, the response of the plant can be controlled so that it will react to inputs in a desired manner in spite of

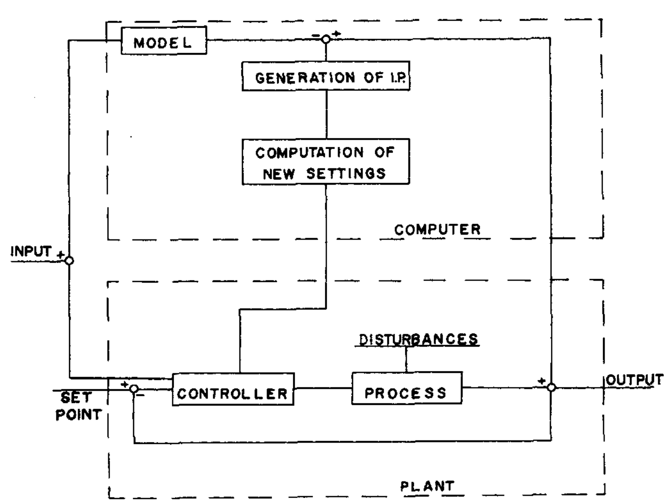

Fig. 1. Block diagram for model reference adaptive control system.

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other loads and disturbances. Dynamic models can be subdivided into two classes. One would result from attempting to maintain the original dynamics of the system to insure that ordinary controllers are operating on the system for which they were designed. A second dynamic model could be developed by attempting to maintain certain dynamic characteristics, such as gain, in order to maintain known responses to known inputs in the face of unknown disturbances. In a sense, adaptive control minimizes the sensitivity of the system to undesirable loads and disturbances.

Implementation of adaptive control is inherently dependent on the means used to compare the system and the model. A common method of comparing the two is to define an integral quadratic index of performance (*12*) as

$$\mathrm{IP} = \int_{t_1}^{t_2} \mathbf{e}^T C \mathbf{e} \, dt \tag{1}$$

where C is a positive semidefinite upper triangular matrix and $\mathbf{e}$ is a state vector describing the difference between the plant and the model. Minimization of this IP with respect to controllable parameters has the effect of minimizing the deviations of the plant outputs from desired values.

PREVIOUS WORK

Although adaptive control is rapidly being recognized for its theoretical value, few papers are available in the literature concerning adaptive control of chemical processes. Among recent publications, however, a few are of particular interest. Crandall and Stevens (*2*) controlled a process identical to the one used in this work with the exception that a single parameter controller was used instead of the two- and three-mode controllers used here. The technique depended upon a decision process that involved a direct search for the controller setting which minimized

$$\mathrm{IP} = \int_0^t e^2 \, dt \tag{2}$$

A limitation was that a multivariable search of an integral expression for more than one parmeter setting could be time consuming and imprecise. Furthermore, the use of an integral error squared IP of this type can result in physically unrealizable controller settings (*10*).

Casciano and Staffin (*1*) controlled a slightly less complex system, but again only one parameter was adaptively controlled. Their technique involved minimizing a Lyapunov function for the system by applying a correction signal to the controller setting rather than calculating its actual value.

Stevens and Wanninger (*13*) controlled a system similar to the one used here, in which forcing functions (as opposed to adaptive control of dynamic characteristics) were determined through use of Lyapunov functions. This technique involved determination of coefficients which not only satisfied Lyapunov's criteria for stability but also drove the system to be as "stable" as possible. In the face of unpredictable disturbances, this approach may no longer be entirely satisfactory. Recently a technique for adaptively controlling a two-parameter system has been demonstrated (*5*), but unfortunately two criteria of performance must be applied to the system.

Parks (*11*) derived an adaptive control scheme based on stability determined from Lyapunov functions. The scheme allows the determination of instantaneous values of the derivative of time-varying controllable parameters. Included in the Lyapunov function is a term involving the deviation of plant parameters from model parameters, along with the deviations of plant output from model output. This technique omits the concept of a designer-chosen IP and also requires that the plant and model be of the same order.

PRESENT METHOD

In this work a scheme is proposed which, after being derived for a system of a given order, may be used to control adaptively any number of parameters, for any system of that order, provided the means are available to search an algebraic expression in multidimensional space or to solve as many nonlinear simultaneous algebraic equations as there are parameters to be controlled. In the earlier work, second-order systems were usually considered and some work was done on extension to higher orders. Here a third-order plant is considered, and adaptive control is exercised over two or three of its parameters. The method developed is applicable to any order plant.

Certain conditions must be met in order for this technique to be effective.

(a) An explicit function of certain parameters (derived later) must have a minimum with respect to those parameters.

(b) Differential equations, which at least approximate the behavior of the plant, must be available which contain the effects of the controllable parameters.

If condition (a) is satisfied, the minimum may be found by any of the classical techniques available for the minimization of a function. The conditions in (b) may be satisfied in many ways, the most desirable being exact knowledge of the system equations, variables, and parameters at all times. If this is not possible, as is generally the case, reasonable approximations to the system equations or parameters or both may be used. Depending, of course, on the accuracy of the approximations, significant improvement of the IP over that of the unadapted system index of performance can be obtained. Accuracy will be a particular problem in attempting to control plant start-up by use of linearized versions of the system equations, for the deviations from steady state will be significant.

DEVELOPMENT

The method developed here allows the minimization of a popular form of integral IP by expressing the integral IP as an algebraic function of controllable parameters. This function can be minimized by classical techniques, such as the setting of its derivatives with respect to the controllable parameters equal to zero, solving for the extreme point, and assuring that the point is a minimum by checking second derivatives. A search technique might also be used, but even in this case the computation can be greatly simplified in the algebraic approach when the integrated functional form is known.

The method assumes that the form of the differential equations which describe the system are known, or that the effects of controllable parameters are included in a linear differential equation which approximates the system. On the basis of this assumption, a control law can be derived which is valid for any system of a given order.

Suppose a system of homogeneous linear differential equations is available in the form

$$\dot{e}_i = \sum_{k=1}^{n} b_{ik} e_k \quad i = 1, \cdots, n \tag{3}$$

where $\mathbf{e}$ is a state vector for the n-dimensional system and b_{ik} is the k^{th} coefficient in the i^{th} equation. An objective function

$$V = \sum_{i=1}^{n} \sum_{j=i}^{n} a_{ij} \, e_i \, e_j \quad a_{ij} = 0, \quad i > j \tag{4}$$

can be defined, where n is the number of state variables required to specify the system, the a_{ij}'s are undetermined coefficients, and $\mathbf{e}$ is the state vector for the system. Taking the derivative of V with respect to the independent variable of the system (usually time)

$$\dot{V} = \sum_{i=1}^{n} \sum_{k=1}^{n} e_i \, e_k \left[\sum_{j=1}^{i} a_{ji} \, b_{jk} + \sum_{j=i}^{n} a_{ij} \, b_{jk} \right] \quad (5)$$

Now, a positive semidefinite quadratic function of the state variables can be chosen, such as $F(\mathbf{e}) = \mathbf{e}^T C \mathbf{e}$, where C is a weighting matrix of order n whose diagonal entries are normally 1.0 from c_{11} to c_{mm} and whose other elements are normally 0, and $m \leq n$. An index of performance can be defined in terms of this function as

$$\text{IP} = \int_{t_1}^{t_2} F(\mathbf{e}) \, dt \quad (6)$$

where t is the independent variable of the system. Since F is a positive semidefinite quadratic function and $\dot{V}$ is a general quadratic form having unknown coefficients, $\dot{V}$ can be set equal to F. This equality will determine the coefficients a_{ij} in $\dot{V}$. The set of algebraic equations which results is as follows.

$$\text{For} \quad i = k: \quad \sum_{j=1}^{i} a_{ji} \, b_{ji} + \sum_{j=i}^{n} a_{ij} \, b_{ji} = c_{ii} \quad (7)$$

Also, as $c_i \, e_k = e_k \, e_i$ and C is an upper triangular matrix,

$$\text{for} \quad i < k: \quad \sum_{j=1}^{i} a_{ji} \, b_{jk} + \sum_{j=1}^{k} a_{jk} \, b_{ji} + \sum_{j=i}^{n} a_{ij} \, b_{jk} + \sum_{j=k}^{n} a_{kj} \, b_{ji} = c_{ik} \quad (8)$$

where $c_{ik} = 0$ if F is chosen with no cross terms such as $c_{ik} \, e_i \, e_k$. There are now $(n^2 + n)/2$ equations in $(n^2 + n)/2$ a_{ij}'s. These equations can be solved for the a_{ij}'s as functions of the b_{ij}'s and the C matrix, resulting in

$$a_{ij} = f_{ij} \, (B, C) \quad (9)$$

where B is the $n \times n$ matrix (b_{ij}).

Since $\dot{V}(\mathbf{e}) = F(\mathbf{e})$, it follows that

$$\text{IP} = \int_{t_1}^{t_2} \dot{V}(\mathbf{e}) \, dt \quad (10)$$

and

$$\int_{t_1}^{t_2} \dot{V}(\mathbf{e}) \, dt = V(\mathbf{e}) \Big|_{t_1}^{t_2} \quad (11)$$

Thus,

$$\text{IP} = V(e) \Big|_{t_1}^{t_2} \quad (12)$$

In effect, the manipulations of Equations (4) through (12) consist of defining an algebraic objective function (V) with unknown coefficients, constraining it to be equal to the integrand of a classical integral index of performance (IP) and determining the coefficients of V. The net result is that the usual integral objective function can be replaced by a complex but more manageable algebraic objective function, as expressed by Equation (12).

In order to simplify the solution to Equations (7) and (8) and also to include the effects of the model, the state vector for the system was chosen to be a particular system variable and its $n - 1$ derivatives. The system variable chosen was the deviation of the plant output from the model output. The effect of this is to make the matrix B a matrix whose first $n - 1$ rows are zero except for 1.0's in the location immediately to the right of the diagonal element in these rows, and whose n^{th} row is the vector

$$\mathbf{B} = (B_1, B_2, B_3, \cdots B_n)$$

For use in Equation (9), the elements of the B-matrix become

$$b_{jk} = \begin{cases} 1, & k = j + 1 \text{ for } j = 1, n - 1 \\ 0, & k \neq j + 1 \text{ for } j = 1, n - 1 \\ B_k, & j = n \end{cases} \quad (13)$$

Combining Equations (4), (9), and (12),

$$\text{IP} = \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} \, (B, C) \, [e_i e_j|_{t_2} - e_i e_j|_{t_1}] \quad (14)$$

Because B is a known (or approximately known) function of the controllable parameters $\boldsymbol{\alpha}$, the IP may now be minimized with respect to any number of α_i's by well-known classical minimization techniques if the values of $\mathbf{e}(t_2)$ and $\mathbf{e}(t_1)$ are known or can be reasonably approximated. Thus, t_1 and t_2 must be specified and the values of $\mathbf{e}$ determined at these points. This can be done by letting t_1 be the current time t (or the time at which an updating of $\boldsymbol{\alpha}$ is to occur), at which point $\mathbf{e}$ is known, and choosing t_2 to be infinite. Assuming that any models chosen for model reference adaptive control will be stable, if we can determine the stability requirements for the plant and assure their satisfaction at all times, it is then obvious that $V|_\infty = 0$ since $\mathbf{e}|_\infty = \mathbf{0}$. Thus

$$\text{IP} = - V(t_1) \quad (15)$$

It is possible to determine the stability limits on the controller parameters very simply through application of the Routh-Hurwicz criterion to the system equations.

It should be pointed out that in this method we determined those values of the controllable parameters which will optimize system performance from the time of the adjustment to infinity, assuming that the plant will not change from that time forward. If the system were in fact to remain unchanged (at its optimum), these controllable parameters would not be changed at all at subsequent adjustment intervals. If, however, the plant does vary, the parameter settings computed at the next adjustment time would be different, but would again represent the best dynamics the system could have from that moment to infinity.

For a set of nonlinear system equations, the choice of V will determine the form of $\dot{V}$. If a V function can be found for which $\dot{V}$ can be constrained to a satisfactory form for use in Equation (10) and if the number of independent constraint equations is equal to the number of constants a_{ij} used in V, the derivation of the adaptive control law is the same as in the linear case. If a Lyapunov function is known for the system, it may be used for V. If, however, a satisfactory V function is not found for the nonlinear equations, the system can be linearized about an operating point and the method can then be applied to the linearized equations. Unfortunately, this may be a limitation on the efficiency of adaptive control of nonlinear systems using constrained objective functions.

Applications of this method lie in (a) set-point control where the set-point may or may not be the steady state point of the free system, (b) automatic control of plant start-up and shut-down procedures (initial offset problem), and (c) model reference adaptive control where the model may or may not be of the same order as the system. In each case, the system parameters may vary in an unpredictable and often unmeasurable manner.

APPLICATIONS

The optimal way to verify the utility of any control scheme is to set it up to control a real plant. Unfortunately, this is usually impossible so that most researchers resort to computer simulations. This work is no exception; however, we have attempted throughout to keep the applications examples as close to the control of a real process in real time as possible.

The method as outlined has been applied in several ways to the reacting plant described by Kermode and Stevens (6). All numerical data for the process are the same as those given in (6). The plant is a continuous stirred tank reactor in which a first-order exothermic reaction whose rate constant is an exponential function of temperature takes place. Heat is removed by cooling water whose flow rate is controlled by an ordinary (e.g., proportional or proportional-plus-integral) controller. The output of the plant is compared to the output of a model and the controller constants are adjusted according to the previously defined control scheme.

The nonlinear plant equations are

$$\dot{x} = \frac{q}{v}(x_I - x) - kx \qquad (16)$$

$$\dot{T} = \frac{q}{v}(T_I - T) - \frac{(-\Delta H)\,kx}{\rho C_p} - \frac{UA'(T - T_c)}{v\rho C_p(1 + 1/F')} \qquad (17)$$

where $k = A\,e^{-E/RT}$ and $1/F' = \dfrac{UA'}{2\,\rho_c C_{pc} Q_c}$

These equations were linearized about the steady state operating condition and the linear equations were written as

$$\dot{\overline{x}} = K_1\overline{x} + K_2 T \qquad (18)$$

$$\dot{\overline{T}} = K_4\overline{x} + K_5\overline{T} + K_6\overline{Q}_c \qquad (19)$$

The performance index chosen was

$$\text{IP} = \int_{t_1}^{\infty} (C_{11}\,e_1^2 + C_{22}\,e_2^2 + C_{33}\,e_3^2)\,dt \qquad (20)$$

where **e** is the deviation variable described above and $C_{11} = 1$, $C_{22} = C_{33} = 0$ unless otherwise specified, and t_1 is the current time.

In all cases the Univac 1107 was used for three separate purposes. The nonlinear plant equations were integrated numerically to simulate the operation of the plant for both the adapted and unadapted systems. The two plants were simulated by the same program for purposes of comparison. In all cases the same upsets and disturbances were given to both plants. Secondly, the inputs to the plant were used as inputs to the model equations, and these equations were numerically integrated (for dynamic models) to generate the model output. Thirdly, the IP was minimized by numerical solution of

$$\frac{\partial \text{IP}}{\partial \alpha_i} = 0 \qquad (21)$$

or by searching $-V|_{t_1}$ as a function of $\boldsymbol{\alpha}$ for its minimum. The resulting updated controller settings were returned to the adaptively controlled plant for use during the next time interval.

Development of Model

The cooling water flow rate in Equation (19) was controlled by a proportional-plus-integral controller or a three-mode controller acting on the difference between the output concentration and a setpoint which was set to the steady state operating point of the uncontrolled plant with a steady state cooling water flow rate of 0.2 lb./sec. The setpoint could have been taken at a different value with the understanding that the cooling water rate at steady state under these conditions would be different from 0.2 lb./sec. These types of control action result in a third-order closed loop plant in the controlled variable, concentration.

Because second-order behavior of a system of this type is often desirable, the model in Figure 1 was specified as second order. The solutions to the third-order linearized closed loop plant equations were generated on an analog computer along with the second-order model equations. A second-order model was chosen after visual inspection of several second-order solutions to determine the one which behaved most like the undisturbed plant. With the undisturbed plant data as given in Table 1, the model equations used were

$$\dot{x}_m = y_m + D_m \qquad (22)$$

$$\dot{y}_m = -\,.00316\,y_m - .0000196\,x_m \qquad (23)$$

where D_m is zero when the disturbances in the plant do not affect the model. Solutions to these model equations and the undisturbed plant equations were again generated, this time on the digital computer, and the values of the controllable plant parameters were refined, based on minimizing a quality index, until the solutions were as close as possible to one another. This resulted in the controller settings which were optimal for the unadapted plant.

Quality Index

Because the IP chosen for this work represents a prediction into the future, some method was needed to evaluate the performance of the adaptive control scheme. This was done by evaluating a quality index, defined as

$$\text{QI} = \int_0^t F(\mathbf{e})\,dt \qquad (24)$$

for both the unadapted and adapted systems. After both plants have been in operation for a while, a lower QI for a given system indicates that in that system the plant has been behaving more like the model than the plant in the other system.

Systems Studied

PI *Controller*. In this first case, the cooling water was controlled by a PI controller of the form

$$\overline{Q}_c = -\,K_p\overline{x} - K_I\int_0^t \overline{x}\,dt \qquad (25)$$

TABLE 1. STEADY STATE DATA

Quantity	Steady state value
A	3×10^{11} (sec.$^{-1}$)
A'	500 (ft.^{2})
C_c	1 (B.t.u./lb. °R.)
C_p	1 (B.t.u./lb. °R.)
E	45,000 (B.t.u./lb.-mole)
ΔH	−20,000 (B.t.u./lb.-mole)
q	0.5 (cu. ft./sec.)
Q_c	0.2 (cu. ft./sec.)
R	1.987 (B.t.u./lb.-mole °R.)
T	660.8 (°R.)
T_I	690 (°R.)
T_c	520 (°R.)
U	100 (B.t.u./hr.-sq. ft. °R.)
v	100 (cu. ft.)
x	0.463659 (lb.-mole/cu. ft.)
x_I	0.5 (lb.-mole/cu. ft.)
ρ	60 (lb./cu. ft.)
ρ_c	60 (lb./cu. ft.)

For the adaptive feature, the second-order model mentioned was chosen. The system equations were specified as follows. Let

$$e_1 = \bar{x} - \bar{x}_m \tag{26}$$

Combining the linearized system Equations (18) and (19), Equation (25) derived for $\bar{Q}_c$, and the model Equations (22) and (23), the system equations can be written as

$$\dot{e}_5 = B_1 e_1 + B_2 e_2 + B_3 e_3 + B_4 e_4 + B_5 e_5 \tag{27}$$

where the state vector is

$$\begin{aligned} e_1 &= e_1 \\ e_2 &= \dot{e}_1 \\ e_3 &= \ddot{e}_1 \\ e_4 &= e_1^{(3)} \\ e_5 &= e_1^{(4)} \end{aligned} \tag{28}$$

and

$$\begin{aligned} B_1 &= -BB_1 K_{1m} \\ B_2 &= -BB_1 K_{2m} - BB_2 K_{1m} \\ B_3 &= BB_1 - BB_2 K_{2m} - BB_3 K_{1m} \\ B_4 &= BB_2 - BB_3 K_{2m} + K_{1m} \\ B_5 &= BB_3 + K_{2m} \end{aligned} \tag{29}$$

where

$$\begin{aligned} BB_1 &= K_1 + K_5 \\ BB_2 &= K_2K_4 - K_1K_5 - K_2K_6K_p \\ BB_3 &= -K_2K_6K_I \\ K_{1m} &= -.0000196 \\ K_{2m} &= -.00316 \end{aligned} \tag{30}$$

The set of Equations (7) and (8) were solved analytically for the a_{ij}'s using $n = 5$. These solutions were programmed and the resulting IP, Equation (15), was minimized with respect to the controller settings K_p and K_I. In the unadapted system, which was solved for comparison purposes, the best constant values for K_p and for K_I as determined by quality index considerations above were used.

Three Mode Controller. In the second case, the cooling water was controlled by a three-mode controller of the form

$$\bar{Q}_c = -K_p\bar{x} - K_I \int_0^t \bar{x}\,dt - K_D \frac{d\bar{x}}{dt} \tag{31}$$

The addition of derivative control does not increase the order of the plant equations, the only difference between this case and the first thus being the addition of K_D to some terms in the **B**-vector. The resulting IP was minimized with respect to all three controller settings.

Minimization Processes

Three different minimization techniques were employed. The first was a direct search of the IP function using sectioning (*14*) in multivariable search. The second and third methods involved application of Newton-Raphson root-finding procedures to the derivatives of the IP with respect to the controller settings. In both cases the derivatives of the IP were generated as follows.

Because

$$\mathrm{IP} = -\sum_{i=1}^{n}\sum_{j=i}^{n} a_{ij}\, e_i e_j\big|_{t_1} \tag{32}$$

and the a_{ij}'s are known functions of C and B, and B in turn is a function of the controller settings, $\boldsymbol{\alpha}$, then

$$\frac{\partial \mathrm{IP}}{\partial \alpha_m} = \sum_{i=1}^{n}\sum_{j=i}^{n}\left[\sum_{s=1}^{n} \frac{\partial a_{ij}}{\partial B_s}\frac{\partial B_s}{\partial \alpha_m}\, e_i e_j\big|_{t_1,\,\alpha_m} + \frac{a_{ij}\,(e_i e_j|_{t_1,\,\alpha_m+\Delta\alpha_m} - e_i e_j|_{t_1,\,\alpha_m})}{\Delta\alpha_m}\right] \tag{33}$$

The **e** vector at time t_1 can be evaluated by knowing the system variables (concentration, temperature, model concentration, etc.) at time t_1 and the controller settings and by evaluating the intermediate equations found in deriving the state vector. This eliminates the problem of having to measure derivatives in the physical plant.

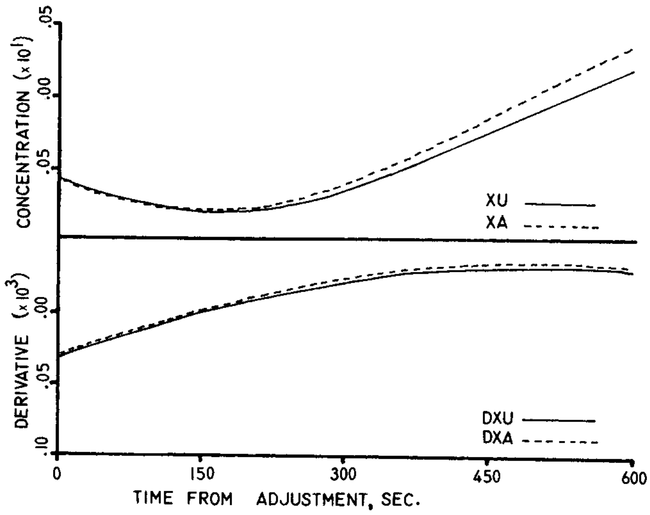

Fig. 2. Deviations from steady state values of concentration and its derivative for adapted and unadapted systems to demonstrate the effect of controller parameter adjustments.

Using these derivatives [Equation (33)], the second minimization method involved the direct application of the Newton-Raphson root-finding technique for n-dimensional equations (n being the number of controllable parameters α_i). The third method was a modification of the Newton-Raphson approach in that the new controller settings were determined by application of the Newton-Raphson technique, but these new settings were not used if they resulted in a higher value for the IP. This is similar to a root-finding modification proposed by Lance (*7*). At no time during the application of any of the minimization methods were any of the controller settings allowed to violate the conditions for the stability of the plant, as determined by application of the Routh-Hurwicz criterion to the linearized equations for the plant.

Identification

The problem of identifying the state of the system was not investigated in this work, but the effects of identification were examined. Since the **e** vector here is evaluated from the linear equations, any identification based on these linearized equations is inherently imperfect. Also because the IP is a function of the coefficients in the linear system (which are in turn variable as system parameters change), some runs were made in pairs, one run using the steady state values of these coefficients (corresponding to no identification) and the other including the effects of parameter variations on these coefficients. The results of these comparisons, showing the effects of perfect identification (or lack of identification), are presented; of course, there is an implicit assumption here that the system never ventures extraordinarily far from its steady state.

Time Scaling

Due to the magnitude of the coefficients in the linear equations, it was necessary to apply the technique of time scaling to the coefficients in the resulting system equations in terms of the state vector. The Univac 1107 can handle numbers no smaller than 10^{-37}, and use of the original coefficients resulted in numbers lower than that being generated by the computations involved in the control scheme. Time scaling eliminated this problem.

Time Interval between Parameter Adjustments

It was found that the step changes in controller settings introduced by adaptive control did not affect the system significantly until a certain amount of time had passed. If another controller setting adjustment was attempted before the system had really experienced the effects of the previous adjustment, the adapted system performance could be expected to be worse in some cases than the unadapted system performance. In order to approximate the time interval over which the system was not fully affected by a controller adjustemnt, the following analysis was used.

If K_p and K_I are considered functions of time, Equation (25) for $\dot{\overline{Q}}_c$ can be linearized about some point in time t as

$$\dot{\overline{Q}}_c = (K_p|_t - K_p)\dot{\overline{x}}|_t + (K_I|_t - K_I)\ \overline{x}\,|_t - K_p|_t\,\dot{\overline{x}} - K_I|_t\,\overline{x} \quad (34)$$

Combining this equation with Equations (18) and (19) results in

$$\dddot{\overline{x}} - (K_1 + K_5)\ \ddot{\overline{x}} - (K_2K_4 - K_1K_5 - K_2K_6K_p|_t)\ \dot{\overline{x}} + K_2K_6K_I|_t\ \overline{x} = K_2K_6\dot{\overline{x}}\,|_t\ (K_p|_t - K_p) + K_2K_6\overline{x}|_t\ (K_I|_t - K_I) \quad (35)$$

It can readily be seen that, if there is no change in either K_p or K_I, this is a homogeneous differential equation describing the behavior of $\overline{x}$. A step change in either K_p or K_I (or both) results in a nonhomogeneous differential equation which has a constant forcing function. The effect of introducing this forcing function may be examined by solving Equation (35) both with and without the forcing function, both solutions beginning with the same initial conditions. A typical set of data was chosen and these solutions were generated, once assuming a parameter adjustment had not been made and once with the effect of the adjustment included. These solutions are presented in Figure 2.

It can be seen that the effect of the adjustment is not significant in the $\overline{x}$ variable until about 300 or 400 sec. have elapsed, although a significant change in $\dot{\overline{x}}$ is seen as early as 200 sec. after the adjustment. On this basis, it was decided that an adjustment interval of 400 sec. was sufficient to allow the effect of the adjustment to be felt. Since the **e** vector has only partial information about the previous adjustment before about 400 seconds has elapsed, the predicted IP would be biased by this lack of information if a new adjustment were attempted, and the system could not be expected to perform ideally. Thus, 400 sec. was used as the time interval between parameter adjustments in generating all the results presented below.

Disturbances

Three distinct types of disturbances were used to demonstrate the versatility of the adaptive scheme. The first type gave both the plant and the model the same initial conditions on concentration and its derivative to simulate the problem of plant start-up. This disturbance is equivalent to the problem of a change in set-point. The second type of disturbance allowed variations in time in some of the plant parameters which did not affect the model, thus simulating processes in which time-varying parameters might have a deleterious effect on plant performance. The third type of disturbance used was the inclusion of changes

Table 2. List of Cases Studied

Case	Controller	Adapted parameters	Disturbances†	Decision**	Identification	QI adapted $\times 10^{-2}$	QI unadapted $\times 10^{-2}$	%††
1*	PI	K_p	IC	S1	No	4.586	6.541	70.11
2*	PI	K_I	IC	S1	No	3.407	6.541	52.08
3*	PI	K_p,K_I	IC	S2	No	2.914	6.541	44.54
4	PI	K_p	IC	S1	No	3.287	5.989	54.88
5	PI	K_I	IC	S1	No	2.211	5.989	36.91
6	PI	K_p,K_I	IC	S2	No	1.654	5.989	27.61
7	PI	K_p,K_I	IC	MN	No	1.750	5.989	29.22
8	PI	K_p,K_I	1,IC	MN	No	4.039	7.294	55.37
9	PI	K_p,K_I	1,IC	MN	Yes	3.667	7.294	50.27
10	PI	K_p,K_I	1	MN	No	0.0244	0.4359	5.59
11	PI	K_p,K_I	1	MN	Yes	0.1659	0.4359	38.05
12	PI	K_p,K_I	2	MN	No	0.3208	0.5544	57.86
13	PI	K_p,K_I	2	MN	Yes	0.3208	0.5544	57.86
14	PI	K_p,K_I	3	MN	No	0.1637	5.457	2.99
15	PI	K_p,K_I	3	MN	Yes	2.420	5.457	44.34
16	PI	K_p,K_I	4	MN	Yes	0.00083	17.041	0.00
17	PI	K_p,K_I	IC,1,2,3	MN	No	3.275	14.259	22.96
18	PI	K_p,K_I	IC,1,2,3	MN	Yes	2.965	14.259	20.79
19	PI	K_p,K_I	IC,1,2,3	S2	Yes	2.834	14.259	19.87
20	PID	K_D	IC	S2	No	2.793	5.632	49.59
21	PID	K_p,K_I,K_D	IC	S2	No	0.00722	5.632	0.12
22	PID	K_p,K_I,K_D	IC,1,2,3	S2	No	1.278	12.281	10.40
23	PID	K_p,K_I,K_D	IC,1,2,3	S2	Yes	1.370	12.281	11.15
24	PI	K_p,K_I	IC, 1,2,3,5,6,7	S2	Yes	1.652	9.483	17.42
25	PID	K_p,K_I,K_D	IC, 1,2,3,5,6,7	S2	Yes	1.277	8.925	14.30
26	PID	K_p,K_I,K_D	IC, 1,2,3,5,6,7,8	S2	Yes	1.161	8.925	13.00
27	PID	K_p,K_I,K_D	IC, 1,2,3,5,6,7,9,	S2	Yes	3.441	8.925	38.55

* Denotes linear plant equations.

** Decision techniques: S1—univariable search, S2—sectioning, and MN—modified Newton's method.

† Disturbances—IC: $x(0) = 1.05x_s$; 1: $k = k_s\ e^{-.0001t}$; 2: $x_I = x_{IS} + .001 \sin(.003t)$; 3: $T_c = T_{cs} + 10 \sin(.006t)$; 4: $T_c = T_{cs} + 10 \sin(.003t)$; 5: $q = q_s + .001 \sin(.006t)$; 6: $U = U_s\ (1.0 + .00005t)$; 7: $T_I = T_{IS} + 5 \sin(.001t)$; 8: $(c_1 = c_2 = c_3 = 1)$; 9: $c_1 = 1$, $c_2 = 0$, $c_3 = 10^4$.

†† Adapted QI as a percentage of unadapted QI, both evaluated at 8,000 sec.

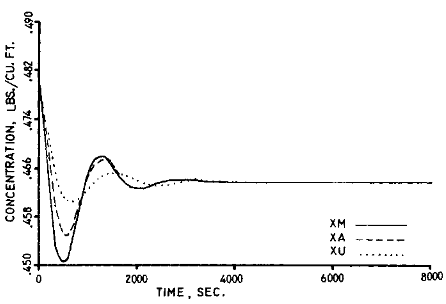

Fig. 3. Case 7: concentration vs. time.

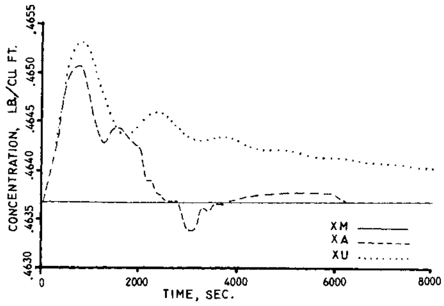

Fig. 4. Case 11: concentration vs. time.

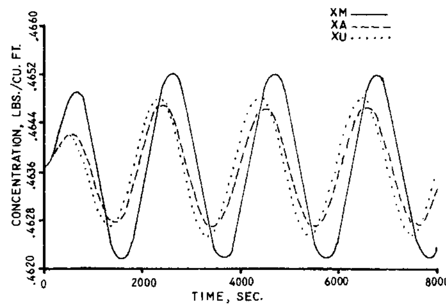

Fig. 5. Case 13: concentration vs. time.

in load variables which affected both the plant and the model to determine the ability of the system to track a dynamic model.

RESULTS

The method for adaptive control via the constrained objective functions was applied to the reacting system described by Kermode and Stevens (6), using either a proportional-plus-integral or a proportional-plus-integral-plus-derivative controller to regulate the cooling water flow rate. The steady state values used for the pertinent variables and parameters are given in Table 1. For the cases in Table 2, the initial condition disturbances were included to simulate plant start-up (or set-point change). Other types of disturbances were investigated, such as catalyst activity decay (which did not affect the behavior of the model) and upsets in the inlet concentrations (which did affect the behavior of the model). Representative examples of the cases in Table 2 are presented in detail in Figures 3 to 10.

DISCUSSION

In all cases studied, significant improvement in performance was obtained by applying adaptive control to the plant. Figure 3 shows that, for the initial offset problem, the adapted system tracks the model far more closely than does the unadapted system, both with regard to amplitude and frequency. The constant controller parameters for the unadapted system were set at their optimal values (which minimized the quality index over time), so the improvement is necessarily due to the ability of the adaptive scheme to determine time-dependent optimal controller settings. The improvement in quality index in this case is 70.78%, even though only two of the three modes of control were used. In this case, as in several others, the parameter values calculated in the first application of adaptation (based on minimizing the IP) were very close to those chosen for the unadapted system (based on minimizing the QI as discussed above).

The ability of the adapted system to alter its own characteristics provides a means of maintaining a high degree of control in the face of unknown or unpredictable disturbances (or both) in the plant. Figures 4 and 5 present the concentration behavior of Cases 11 and 13, respectively, in which the plant was subjected to unpredictable disturbances. In Case 11, the disturbance affects the plant but not the model, so the model remains constant at the set-point throughout. The adaptive control feature was applied for the first time 20 sec. after the disturbance was initiated, and repeated applications every 400 sec. thereafter forced the adapted plant to return to its steady state value much more rapidly than the unadapted system as can be seen in Figure 4. In Case 13, the disturbance affected the model as well as the plant. As shown in Figure 5, the adapted system in this case has forced the plant to follow the frequency of the model at some expense to its amplitude-following characteristics. This resulted in a quality index improvement of 42.14%, as seen in Table 2.

Figures 3, 4, and 5 present results for plants which used PI controllers. Figure 6 presents the concentration curves for Case 21. The conditions for this case are similar to those for Case 7 (Fig. 3), with the exception that a PID controller was used. The additional controllable parameter led to much greater improvement in the quality index, 99.9% compared to 70.78% for Case 7.

Figure 7 presents the concentration curves for Case 27, in which the IP used was

$$\mathrm{IP} = \int_t^\infty (e_1^2 + 10^4 e_3^2)\, dt$$

The addition of the e_3 term allowed the adapted system to track the model frequency in the face of several disturbances better than it did in Case 25, where the same disturbances were present. The reason for this is that the IP being minimized in Case 27 takes into account deviations in direction of the controlled variable as well as deviations in its magnitude. This result is a strong indication that further work should be done to determine an optimal index of performance (which would probably vary with time).

Figures 8, 9, and 10 show detailed results for Case 25. Figure 8 presents the concentration curves similar to those

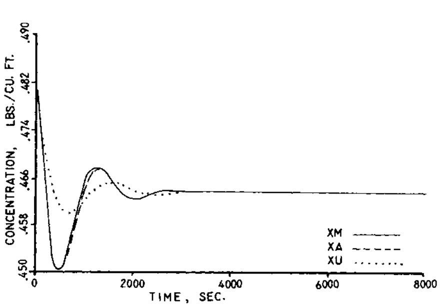

Fig. 6. Case 21: concentration vs. time.

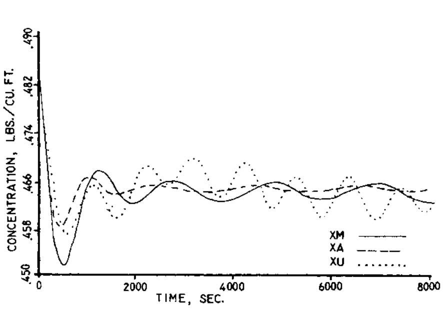

Fig. 7. Case 27: concentration vs. time.

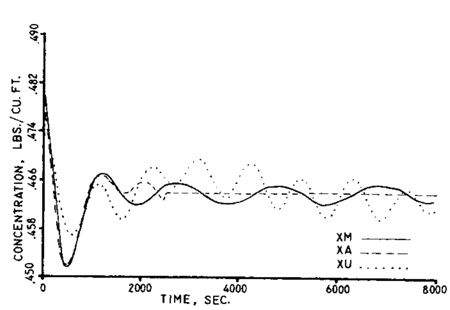

Fig. 8. Case 25: concentration vs. time.

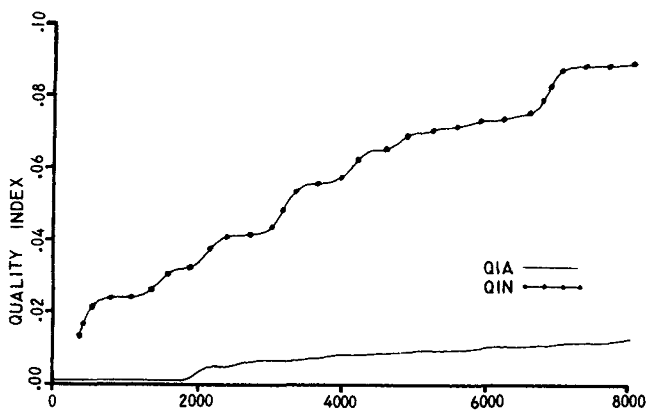

Fig. 9. Case 25: quality index vs. time.

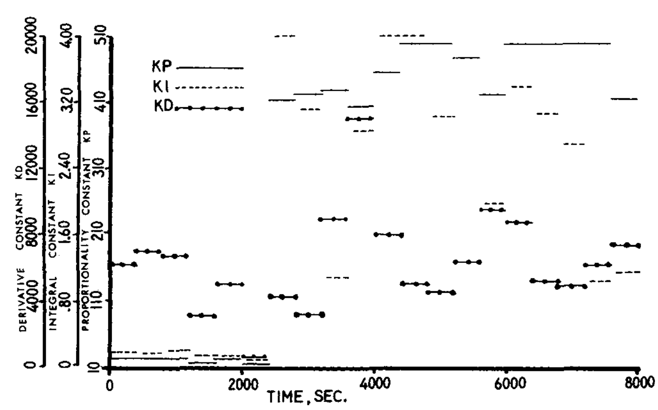

Fig. 10. Case 25: controller coefficients vs. time.

shown in Figures 3 through 7. The concentration in the adapted plant has returned close to its steady state value even though the model is oscillating slightly in response to a disturbance given to it. This still results in a quality index far lower than that in the unadapted system, as can be seen in Figure 9. If tracking of these model oscillations is desired, then the IP should be changed to include terms involving derivatives. The reason for the leveling-off shown in Figure 8 is the high value of the parameter K_P which is calculated after about 250 sec., as seen in Figure 10. This high value has the same effect as a high damping coefficient in a second-order linear system. The other parameters, K_I and K_D, appear to oscillate, but there is no apparent correlation between the frequency of these oscillations and the frequency of the disturbance oscillation.

An important feature of the adaptive control scheme developed here is that it allows the model and the plant to be of different orders. In the applications presented, the addition of integral control raises the plant equations to third order, although the reactor is basically second order. In this case a second-order model can be formulated and used. Naturally, the model response should be chosen to represent as closely as possible some desirable and attainable system response.

Knowledge of the system equations is rather easily obtained in most cases, either through actual knowledge of the governing differential equations or through the techniques of mathematical modeling. Although the actual nonlinear equations were known in this work, the linearized equations were used both in the derivation of the IP equations and in the determination of numerical values for the **e** vector. In the former, the nonlinear equations were not used because a suitable functional form for V could not easily be found. In the latter, the linear equations were used for two reasons. One reason was that only physical variables, such as temperature and concentration, which are relatively easy to measure, were required, as opposed to the necessity of measuring derivatives. The other reason the linear equations were used to determine the **e** vector was to introduce the effects of imperfect knowledge of the plant conditions. The use of the linear equations in these two ways throughout all cases demonstrates the ability of the method to perform under the conditions normally found in real systems.

The effects of identification of the plant were studied by comparing the performance of systems in which the parameter values used in the linear adaptation equations were allowed to change as they did in the nonlinear system equations, as opposed to systems in which the initial values of all parameters were used for adaptation throughout the run. As can be seen in Table 2, this type of identification led to a deterioration in performance when only one disturbance was present (Cases 10 to 15). However, when more than one disturbance was present, this type of identification led to improvement (Cases 8, 9, 17, 18). This behavior is apparently due to differences between the linear and nonlinear systems as they deviate from their steady state values (i.e., due to the fact that perfect identification of the linear approximations to a nonlinear system is still imperfect).

Another improvement over previous adaptive control schemes in this method is that an integral IP may be defined to include not only a variable of interest, but also its derivatives (or, with slight modifications, other variables), as seen in Case 27. This avoids a problem presented by Newton *et al.* (*10*), which shows that the use of integral squared error (ISE) IP's can lead to physically unrealizable or erroneous controller settings. The use of a derivative in the IP balances the attempt to attain a given value for the variable itself against a too rapid approach. In simple cases, the minimum ISE could lead to an infinite derivative and, in more complex cases, to excessive overshoot. In all of the cases in Table 2 (except Cases 26 and 27), the IP was an ISE, so maximum values were arbitrarily chosen for all three controllable parameters. At no time during any of these runs were any of the parameters allowed to exceed their maximum values. Minimum values for all the controllable parameters were determined from the stability requirements.

Of the different decision (or minimization) techniques used, those which involved direct minimization of the IP (search and sectioning search) proved to be slightly better than the use of some form of Newton-Raphson root-finding for the derivatives of the IP. This can be seen by comparing Cases 6 and 7 and Cases 18 and 19. There are two possible reasons for this. One reason is that the second derivatives for use in the root-finding methods were approximated numerically, rather than being calculated analytically. The reason the analytical results were not used, although they are rather easily obtained, is that a considerably longer amount of computer time would have been necessary in each case. Another reason for the slightly poorer performance of the root-finding techniques when applied to more than one controllable parameter was the unsatisfactorily solved problem of what to do if the method predicted values which were outside the allowable ranges. The root-finding procedures did result in lower execution times, which indicated that they found the optimum points faster than the search techniques. For example, on the Univac 1107 the execution time for Case 18 was 4 min., 41 sec., and for Case 19, 6 min., 32 sec. For the three mode controllers, the improvement in performance by the search technique was deemed to outweigh the disadvantage of increased execution time. Even in the most complicated three-mode case, Case 26, the average time required to calculate a new set of parameters was less than 30 sec., which is far smaller than the adjustment interval of 400 sec.

Another significant aspect of this work is that the entire derivation has been obtained in the time domain, thus allowing the outline of the derivation to be applied to nonlinear systems. This application to a nonlinear system involves the difficult choice of a functional form for V, but with luck good results for nonlinear systems may be obtainable.

Because the method derived here results in an algebraic IP which is directly related to an integral expression, the manipulation of integral equations is not necessary. No approximations to integrals are needed to apply the method, nor is there any need for numerical integration in the method itself. This, along with the explicit dependency of the algebraic equations on system parameters, permits elementary techniques for minimization to be applied in order to compute optimal values for controllable parameters.

CONCLUSIONS

Adaptive control using constrained objective functions has been applied to a reacting system and, in the face of various types of disturbances, including the problem of plant start-up, resulted in a significant improvement in plant performance as measured by a quality index.

An algebraic objective function has been derived which, when minimized by any classical analytical or search technique, results in excellent model reference adaptive control.

Identification of the plant parameters has little effect on the performance of the adaptive system in the normal operating regions. This would be true for most real cases, provided deviations are not extremely large.

The method allows the use of a model which is of an order different from that of the plant.

The form of the IP chosen determines the characteristics of the adapted response. Future work in this area could lead to a time-dependent functional form for the optimal IP.

As the V and $\dot{V}$ functions used here are very similar to those that might be used as Lyapunov functions for the system, further work is indicated to attempt also to determine the conditions for stability from the equations themselves, rather than through the use of an independent stability criterion.

ACKNOWLEDGMENT

The authors express their gratitude to the National Science Foundation for a research grant (GK-256) to undertake this work. The work reported is a part of the dissertation of Philip J. Ryan submitted to the Graduate School, University of Notre Dame, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

NOTATION

A = frequency factor in Arrhenius equation
A' = area of cooling coils available for heat transfer
a_{ij} = coefficients in V (Equation 4)
B = matrix of coefficients in state vector equations
$\mathbf{B}$ = vector of coefficients in state vector equation
B_i = elements of $\mathbf{B}$
b_{ij} = elements of B matrix
BB_i = a grouping of terms for use in Equation (29)
C = weighting matrix having elements c_{ij}
C_p = heat capacity.
D_m = a disturbance term added to model Equation (22)
E = activation energy for the reaction
e = difference between system and model outputs
F' = a collection of terms for use in Equation (17)
$F(e)$ = a positive semi-definite function
f_{ij} = functional form of a_{ij}
i, j, k, s = indices for summation
IP = index of performance
K = a constant
K_p, K_I, K_D = proportional and integral and derivative constants for controller
n = order of system differential equations
QI = quality index, Equation (24)
q, Q = flow rate
R = universal gas constant
T = temperature of effluent from tank
t = time
U = heat transfer coefficient
V = a function related to the IP
v = volume of reactor
x = concentration of reacting species
y = derivative of model concentration
α = a generalized set of controllable parameter values
ΔH = heat of reaction $\overset{\Delta H}{(A \rightarrow B)}$
ρ = density

Subscripts

c = cooling water
I = input (except K_I)
m = model
s = steady state

Superscripts

— = deviation from steady state
· = derivative with respect to the independent variable of the system
(n) = n^{th} derivative

Definition of Linear Equation Coefficients

$$K_1 = -\frac{q}{v} - A\exp\left(\frac{-E}{RT_s}\right)$$

$$K_2 = -\frac{x_s E A \exp\left(\frac{-E}{RT_s}\right)}{RT_s^2}$$

$$K_4 = \frac{\Delta H\, A \exp\left(\frac{-E}{RT_s}\right)}{\rho C_p}$$

$$K_5 = -\frac{q}{v} - \frac{UA'}{v\,\rho\, C_p(1. + F'^{-1})} + \frac{x_s E A \exp\left(\frac{-E}{RT_s}\right)\Delta H}{RT_s^2\, \rho C_p}$$

$$K_6 = -\frac{(U A')^2 (T_s - T_c)}{v\,\rho\, C_p\, 2.\, \rho_c\, C_{pc}\, Q_{cs}^2\, (1. + 1./F')^2}$$

Symbols Used in Figures

DXA = derivative of concentration, adapted system
DXU = derivative of concentration, unadapted system
QIA = quality index, adapted system
QIN = quality index, unadapted system
XA = concentration, adapted system
XM = concentration, model
XU = concentration, unadapted system

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Manuscript received June 17, 1968; revision received May 13, 1969; paper accepted May 29, 1969.

Separation of Nitrogen and Methane via Periodic Adsorption

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The periodic process utilizes a rapid pressure swing cycle in an adsorbent bed to effect the separation of gas mixtures. During the first portion of a cycle the compressed gas mixture flows into the adsorbent-filled column. Next, while the feed gas is restrained, an exhaust orifice is opened at the feed end of the column providing depressurization. The product stream is enriched in the component exhibiting the lowest coefficient of adsorption.

A mathematical model based upon the assumption of instantaneous equilibrium between the gas phase and the adsorbed gas was formulated and solved to simulate the periodic, adsorption process. The measured nitrogen content of the product gas stream was found to correlate with the ratio of the product gas rate to the feed gas rate.

At 24°C. the calculated pressure response, feed gas flow rate, and product gas composition correspond favorably with related experimental measurements for all values of the feed gas pressure, cycling frequency, and product gas flow rate within the ranges investigated.

Several unit operations have been studied under controlled cyclic operation since the principle was first reported by Cannon (*1*). Basically, these pulsed operations utilize repetitive parameter changes applied so rapidly that the system behavior remains transient in a cyclic steady state manner. Controlled cyclic operation has produced significant throughput increases and improved efficiencies for processes such as absorption, extraction, and distillation (*2*). In addition, Wilhelm (*3*) successfully demonstrated a dynamic adsorption process for separating fluid mixtures. The process, termed parametric pumping, relies upon an axial thermal gradient induced along a fixed adsorbent bed through which the fluid flows in repeatedly reversed directions.

The dynamic adsorption process to be discussed here differs from parametric pumping in that pressure rather than temperature provides the driving force for the separation. The periodic adsorption process employs a very rapid pressure swing cycle. The compressed feed gas mixture flows into an adsorbent-filled pipe, thereby pressurizing the column. Next, while the feed gas is restrained, an exhaust orifice is opened at the feed end of the column providing depressurization. These feed and exhaust periods of a cycle are of such brief duration (0.05 to 0.4 cycles/sec.) that a product stream may be withdrawn continuously from the opposite end of the pipe. The product stream is enriched in the component exhibiting the lowest coefficient of adsorption.

The objective of this study was to attain a better understanding of the mechanism and behavior of the periodic adsorption process. The practicality of the selected nitrogen-methane feed gas mixture was dictated more by the availability of basic adsorption data (*4*) for such mixtures than by their appreciable natural and commercial occurrence.

Many parameters affect the cyclic separation of the nitrogen-methane mixtures. Of these only the column configuration and the minimum exhaust pressure have remained invariant throughout this investigation. However, the feed gas composition, the type and size of molecular sieve, and the percent feed time per cycle were established at suitable values by preliminary experimentation, and will remain fixed for this discussion. For most experiments, the column contained 42-60 mesh particles of type 5A Molecular Sieve. A nitrogen-methane mixture containing 28.6 mole % nitrogen was fed to the column during 50% of the cycle period. The effects of processing temperature, cycling frequency, feed gas pressure, and product gas flow rate are illustrated and discussed. The experimental results are compared with those predicted by a mathematical model postulating instantaneous equilibrium between the bulk gas and adsorbed gas.

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